

On page 59 of Deep Learning with Python (2nd edition), it is said that $\text{grad}(\text{loss_val}, x2) = 1$. This note answers the question, "Why?"

For the example, $y_true = 4$ (actual value is 4) and $x2 = 7$ (predicted value is 7); so the value of the absolute error loss function is $|4 - 7| = |-3| = 3$.

We can view absolute error as a composition of 3 functions ...

$$\begin{aligned} f(\hat{y}) &= y - \hat{y} \\ g(f(\hat{y})) &= (y - \hat{y})^2 \\ h(g(f(\hat{y}))) &= |y - \hat{y}| = \sqrt{(y - \hat{y})^2} = ((y - \hat{y})^2)^{\frac{1}{2}} \end{aligned}$$

Using the chain rule with $h(g(f(\hat{y})))$, where y_true is replaced with y and $x2$ is replaced by \hat{y} ...

$$\begin{aligned} \frac{\partial |y - \hat{y}|}{\partial \hat{y}} &= \frac{\partial ((y - \hat{y})^2)^{\frac{1}{2}}}{\partial \hat{y}} \\ &= \frac{1}{2} ((y - \hat{y})^2)^{\frac{-1}{2}} * \frac{\partial (y - \hat{y})^2}{\partial \hat{y}} \\ &= \frac{1}{2} ((y - \hat{y})^2)^{\frac{-1}{2}} * 2(y - \hat{y}) * \left(\frac{\partial y}{\partial \hat{y}} - \frac{\partial \hat{y}}{\partial \hat{y}} \right) \\ &= \frac{1}{2} ((y - \hat{y})^2)^{\frac{-1}{2}} * 2(y - \hat{y}) * (0 - 1) \\ &= \frac{1}{2} ((y - \hat{y})^2)^{\frac{-1}{2}} * 2(y - \hat{y}) * -1 \\ &= \frac{1}{2} * 2 * -1 * (y - \hat{y}) * ((y - \hat{y})^2)^{\frac{-1}{2}} \\ &= - \frac{(y - \hat{y})}{((y - \hat{y})^2)^{\frac{1}{2}}} \\ &= - \frac{y - \hat{y}}{|y - \hat{y}|} \end{aligned}$$

For the general case ...

$$\frac{\partial |y - \hat{y}|}{\partial \hat{y}} = \begin{cases} -1 & \text{if } y > \hat{y} \\ 0 & \text{if } y = \hat{y} \\ 1 & \text{if } y < \hat{y} \end{cases}$$